Magnetically coupled circuit

Dr. Mokhtar Said Ibrahim

Introduction

- Electromagnetics is the branch of electrical engineering that deals with the analysis and application of electric and magnetic fields.

- Electromagnetics devices include electric motors and generators, transformers, antennas, radars, microwave ovens and microwave dishes.

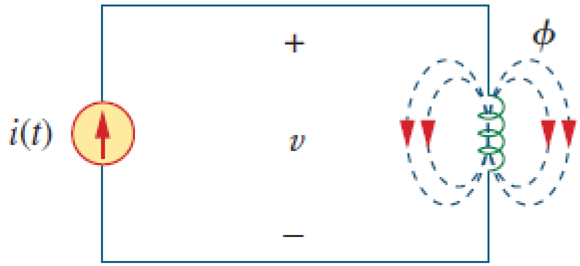
Introduction

- When two loops with or without contacts between them affect each other through the magnetic field generated by one of them, they are said to be *magnetically coupled*.
- The transformer is an electrical device designed on the basis of the concept of magnetic coupling. It uses magnetically coupled coils to transfer energy from one circuit to another.

Mutual Inductance

- When two inductors (or coils) are in a close proximity to each other, the magnetic flux caused by current in one coil links with the other coil, thereby inducing voltage in the latter. This phenomenon is known as *mutual inductance.*
- Let us first consider a single inductor, a coil with N turns. When current i flows through the coil, a magnetic flux is produced around it

Mutual Inductance



- According to Faraday's law, the voltage *v* induced in the coil is:

$$v = N\frac{d\phi}{dt}$$

- But the flux is produced by current *i* so that any change in the flux is caused by a change in the current. Hence,

Mutual Inductance

$$v = N\frac{d\phi}{di}\frac{di}{dt}$$

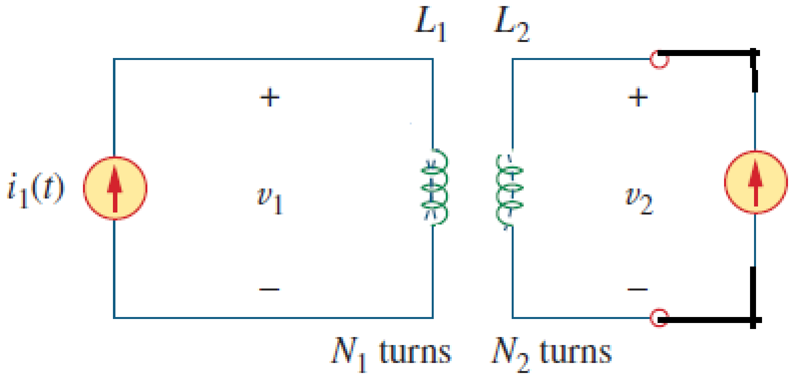
$$v = L\frac{di}{dt}$$

$$L = N\frac{d\phi}{di}$$

- Where L is the self inductance

Mutual Inductance

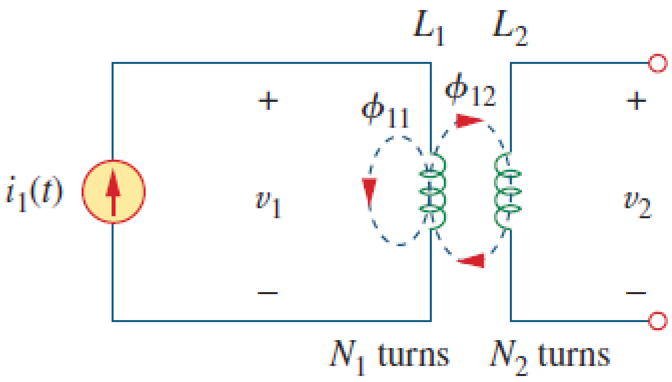
- Now consider two coils with self-inductances and that are in close proximity with each other.



- The analysis solution has two part.

Mutual Inductance

- The first part : assume the second coil is open-circuit



- The total flux in coil 1 is:

$$\phi_1 = \phi_{11} + \phi_{12}$$

Mutual Inductance

- the voltage induced in coil 1 is :

$$v_1 = N_1 \frac{d\phi_1}{dt}$$

$$v_1 = N_1 \frac{d\phi_1}{di_1} \frac{di_1}{dt} = L_1 \frac{di_1}{dt}$$

Where, L1 is the self inductance of coil 1

Mutual Inductance

- the voltage induced in coil 2 is :

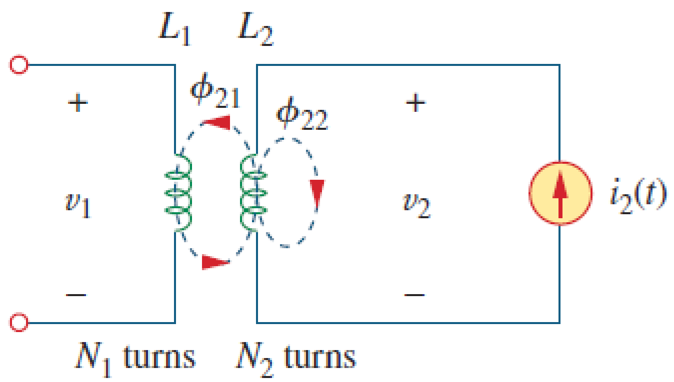
$$v_2 = N_2 \frac{d\phi_{12}}{dt}$$

$$v_2 = N_2 \frac{d\phi_{12}}{di_1} \frac{di_1}{dt} = M_{21} \frac{di_1}{dt}$$

- M21 is known as the *mutual inductance* of coil 2 with respect to coil 1. Subscript 21 indicates that the inductance relates the voltage induced in coil 2 to the current in coil 1.

Mutual Inductance

- The second part : assume the first coil is open-circuit



- The total flux in coil 2 is:

$$\phi_2 = \phi_{21} + \phi_{22}$$

Mutual Inductance

- the voltage induced in coil 2 is :

$$v_2 = N_2\frac{d\phi_2}{dt} = N_2\frac{d\phi_2}{di_2}\frac{di_2}{dt} = L_2\frac{di_2}{dt}$$

$$L_2 = N_2\, d\phi_2/di_2$$

Where, L2 is the self inductance of coil 2

Mutual Inductance

- the voltage induced in coil 1 is :

$$v_1 = N_1 \frac{d\phi_{21}}{dt} = N_1 \frac{d\phi_{21}}{di_2} \frac{di_2}{dt} = M_{12} \frac{di_2}{dt}$$

$$M_{12} = N_1 \frac{d\phi_{21}}{di_2}$$

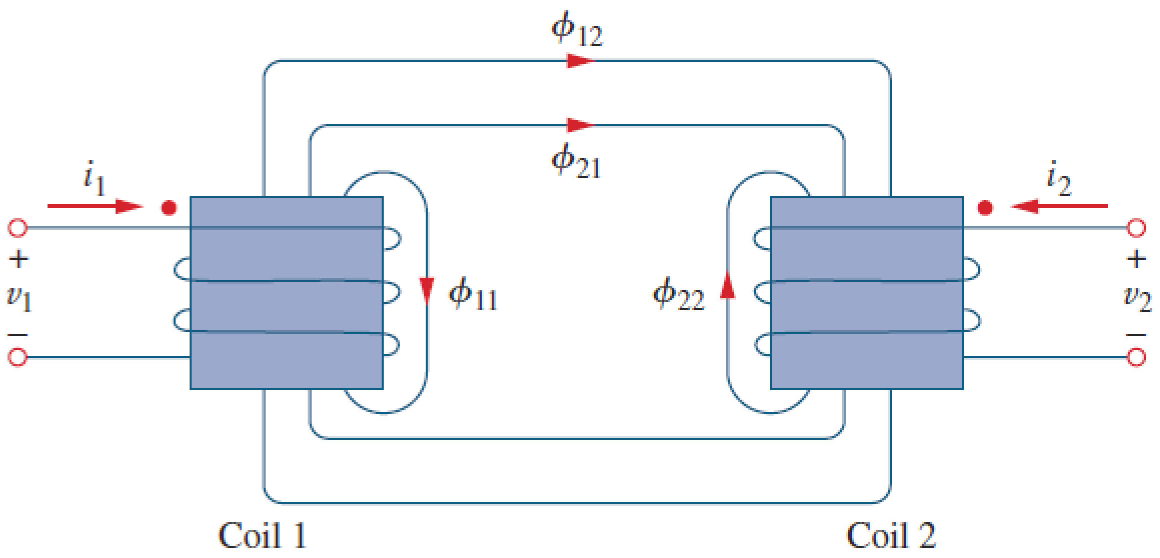
- M12 is known as the *mutual inductance* of coil 1 with respect to coil 2. Subscript 12 indicates that the inductance relates the voltage induced in coil 1 to the current in coil 1.
- Mutual inductance is the ability of one coil to induce a voltage across a neighboring coil.

Dot Convention

- The inductance of coil (L) is always +ve, the polarity of induced voltage on L is determined according to the direction of the current.
- The mutual inductance of coil (M) is always +ve, the polarity of mutual voltage is not easy to determine, because four terminal of two coils are involved.
- so that, dot convention is used to determine the polarity of the mutual voltage.

Dot Convention

- The polarity of the mutual voltage depends on the direction of the current and the dots on the coupled coils.



Dot Convention

- The dot convention is stated as follows:

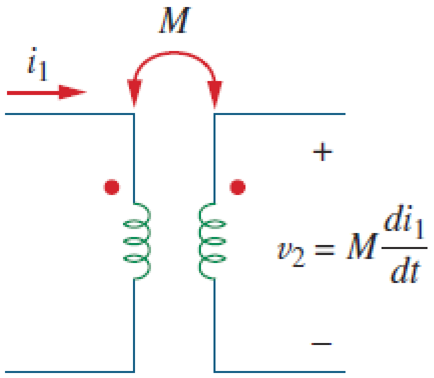
- If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second

- If a current leaves the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is negative at the dotted terminal of the second coil.

Dot Convention

- Application of the dot convention is illustrated in the four pairs of mutually coupled coils.

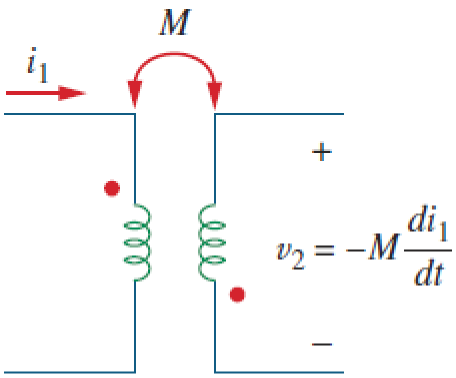
- For the coupled coil in the following figure:
  - I1 enters the dotted terminal of coil 1, V2 is +ve at the dotted terminal of coil 2. so that



$$v_2 = M\frac{di_1}{dt}$$

Dot Convention

- For the coupled coil in the following figure:

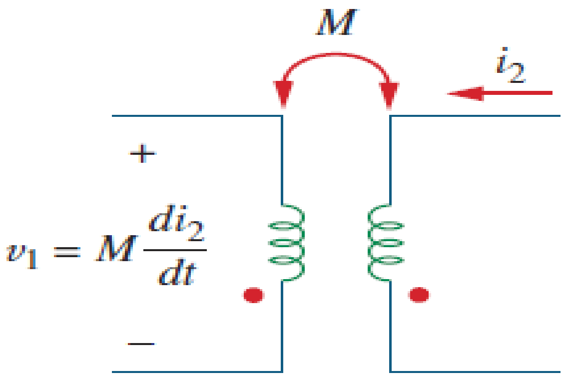


- I1 enters the dotted terminal of coil 1, V2 is -ve at the dotted terminal of coil 2. so that

$$v_2 = -M\frac{di_1}{dt}$$

Dot Convention

- For the coupled coil in the following figure:

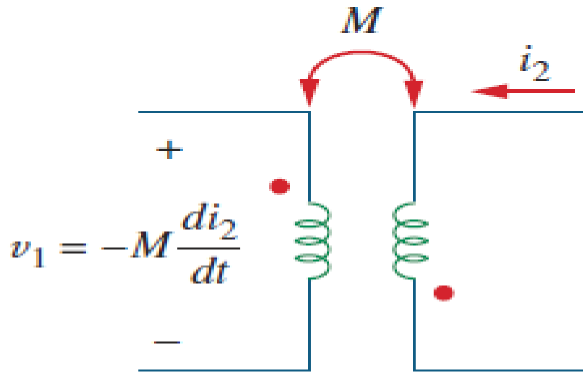


- I2 leaves the dotted terminal of coil 2, V1 is -ve at the dotted terminal of coil 2. so that

$$v_1 = M\frac{di_2}{dt}$$

Dot Convention

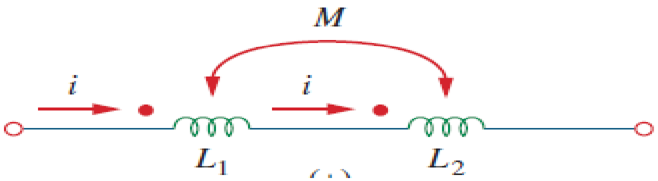
- For the coupled coil in the following figure:



- I2 leaves the dotted terminal of coil 2, V1 is +ve at the dotted terminal of coil 1. so that

$$v_1 = -M\frac{di_2}{dt}$$

Dot Convention for coupled coils in series



$$v_1 = L_1 \frac{di}{dt} + M \frac{di}{dt}$$

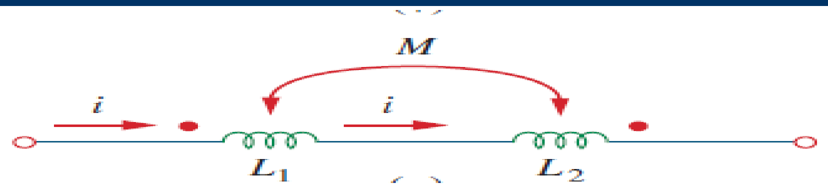
$$v_2 = L_2 \frac{di}{dt} + M \frac{di}{dt}$$

$$v = v_1 + v_2$$
$$= (L_1 + L_2 + 2M) \frac{di}{dt}$$

- The total inductance is :

$$\boxed{L = L_1 + L_2 + 2M \qquad \text{(Series-aiding connection)}}$$

Dot Convention for coupled coils in series



$$v_1 = L_1 \frac{di}{dt} - M \frac{di}{dt}$$

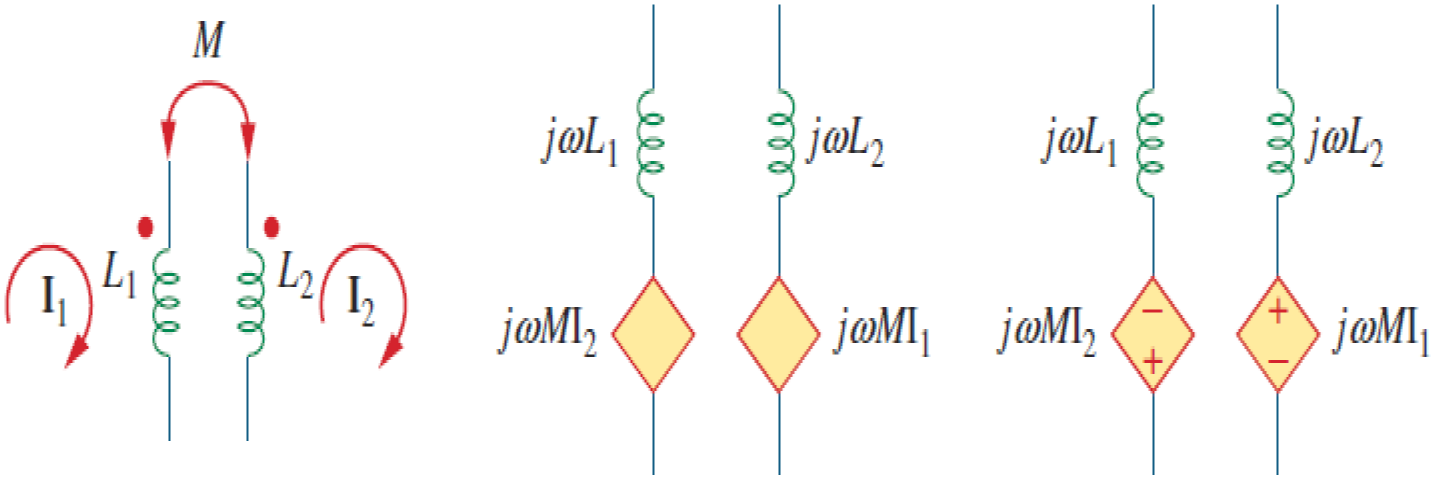
$$v_2 = L_2 \frac{di}{dt} - M \frac{di}{dt}$$

$$\begin{aligned} v &= v_1 + v_2 \\ &= (L_1 + L_2 - 2M) \frac{di}{dt} \end{aligned}$$

- The total inductance is :

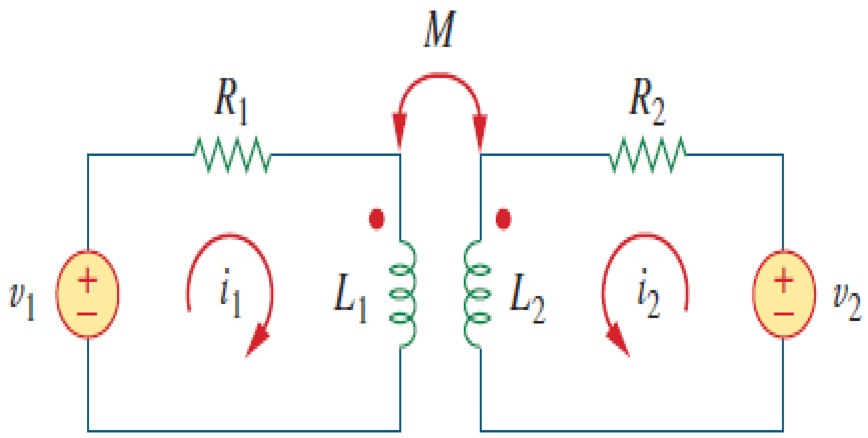
$$\boxed{L = L_1 + L_2 - 2M \qquad \text{(Series-opposing connection)}}$$

Model for Mutually coupled Circuits



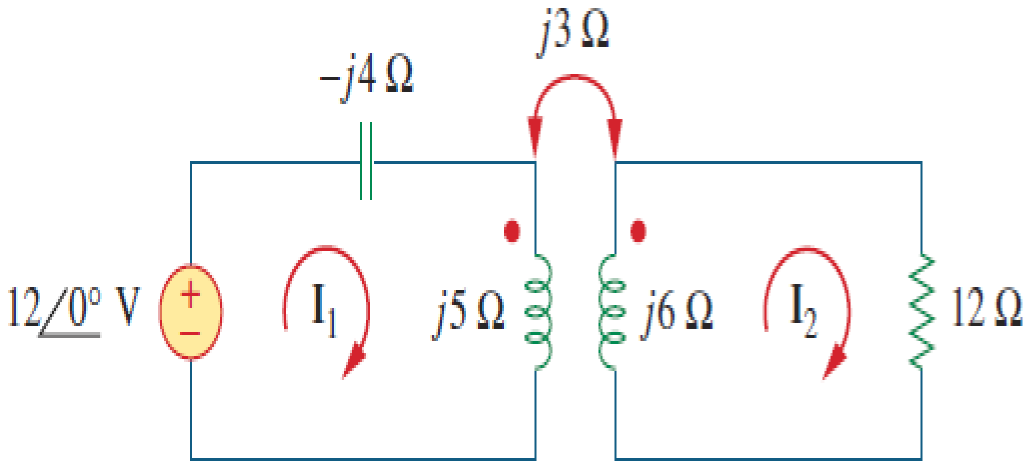
Example

Example1: write the voltage equation for the following circuit.



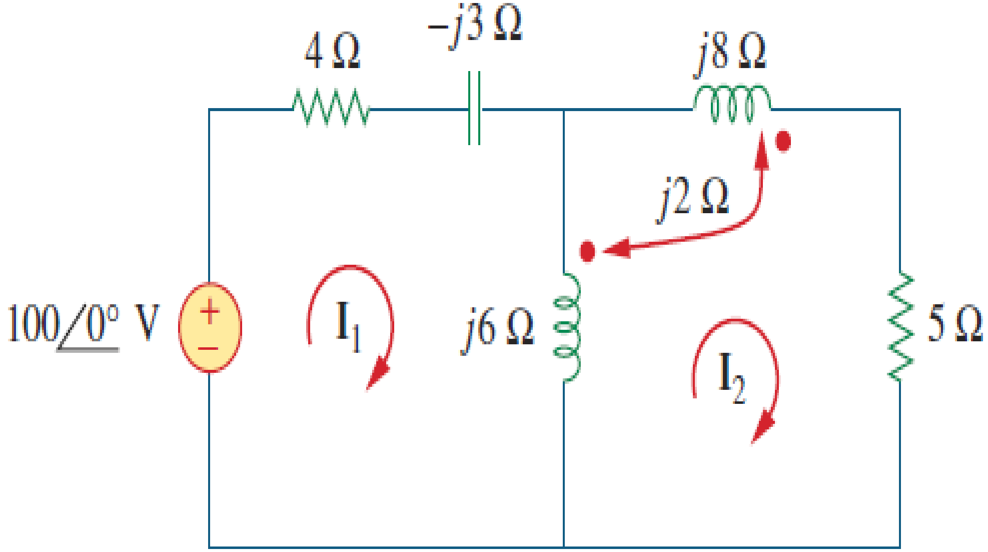
Example

Example2: Find I1 and I2 in the following circuit.



Example

Example3: Find I1 and I2 in the following circuit.



Energy in a coupled circuit

- The energy stored in a coil is given by:

$$W = \frac{1}{2} L i^2$$

- The total energy stored in a two coupled coils, when both currents i1 and i2 have reached constant value are given by:

$$W = \frac{1}{2} L_1 {i_1}^2 + \frac{1}{2} L_2 {i_2}^2 \pm M i_1 i_2$$

- +ve → both i1 and i2 enter or leave the doted terminal of the coil. –ve →otherwise

Energy in a coupled circuit

- The total energy must be greater than or equal zero, so that :

$$\frac{1}{2}L_1{i_1}^2 + \frac{1}{2}L_2{i_2}^2 - Mi_1i_2 \geq 0$$

$$\frac{1}{2}\left(i_1\sqrt{L_1} - i_2\sqrt{L_2}\right)^2 + \left(i_1i_2\left(\sqrt{L_1L_2} - M\right)\right) \geq 0$$

- The square term is positive or zero, so that the second term must be greater than zero

Energy in a coupled circuit

$$\left(\sqrt{L_1 L_2} - M\right) \geq 0$$

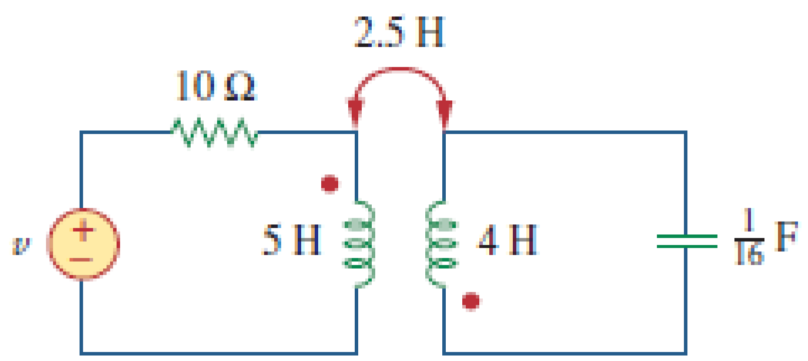
$$\mathrm{M} \leq \sqrt{L_1 L_2}$$

$$\mathrm{M} = k\sqrt{L_1 L_2}$$

- K is the coupling coefficient , is a measure of the magnetic coupling between two coils, $0 \leq k \leq 1$.

Example 4

Consider the circuit in Fig. 13.16. Determine the coupling coefficient. Calculate the energy stored in the coupled inductors at time $t = 1$ s if $v = 60\cos(4t + 30°)$ V.



Linear Transformer

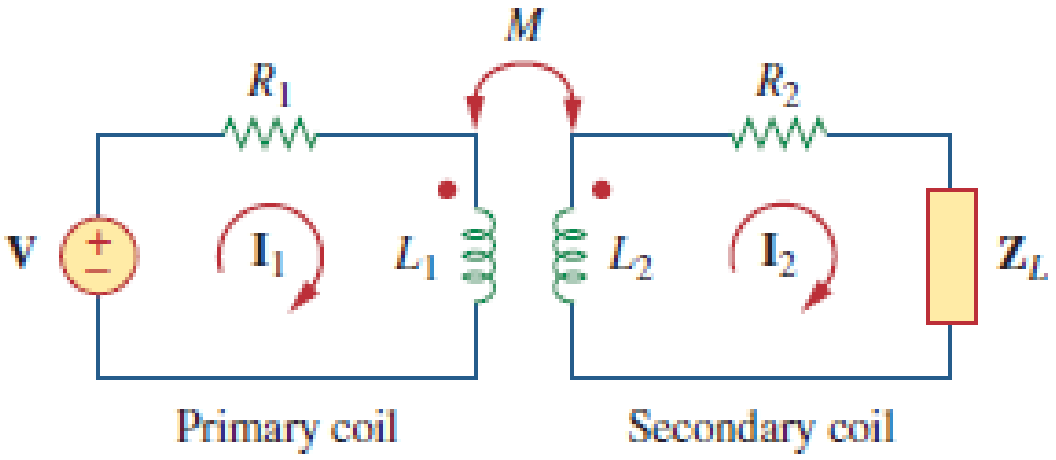
- A transformer is generally a four-terminal device comprising two (or more) magnetically coupled coils.
- The coil that is directly connected to the voltage source is called the *primary winding*. The coil connected to the load is called the *secondary winding*.

Linear Transformer

- The transformer is said to be *linear* if the coils are wound on a magnetically linear material—a material for which the magnetic permeability is constant.
- Linear transformers are sometimes called *air-core transformers.*
- They are used in radio and TV sets.

Linear Transformer

- The equivalent circuit of linear transformer :



Linear Transformer

- Apply KVL for loop 1:

$$\mathbf{V} = (R_1 + j\omega L_1)\mathbf{I}_1 - j\omega M\mathbf{I}_2$$

- Apply KVL for loop 2:

$$0 = -j\omega M\mathbf{I}_1 + (R_2 + j\omega L_2 + \mathbf{Z}_L)\mathbf{I}_2$$

$$I_2 = \frac{jwMI_1}{R_2 + Z_l + jwL_2}$$

- Substitute with I2 in equation of voltage for loop1

Linear Transformer

- The input impedance $Z_{in} = \dfrac{V}{I_1}$

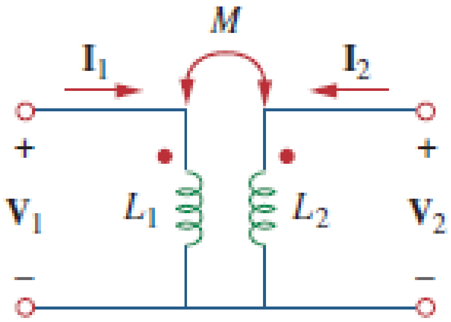
$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}}{\mathbf{I}_1} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}$$

Notice that the input impedance comprises two terms. The first term, $(R_1 + j\omega L_1)$, is the primary impedance. The second term is due to the coupling between the primary and secondary windings. It is as though this impedance is reflected to the primary. Thus, it is known as the *reflected impedance* $\mathbf{Z}_R$, and

$$\boxed{\mathbf{Z}_R = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}} \qquad (13.42)$$

Linear Transformer

- The equivalent circuit of the linear transformer may be replace to T or π $circuit$.
- T circuit:
- The equivalent circuit of the linear transformer.



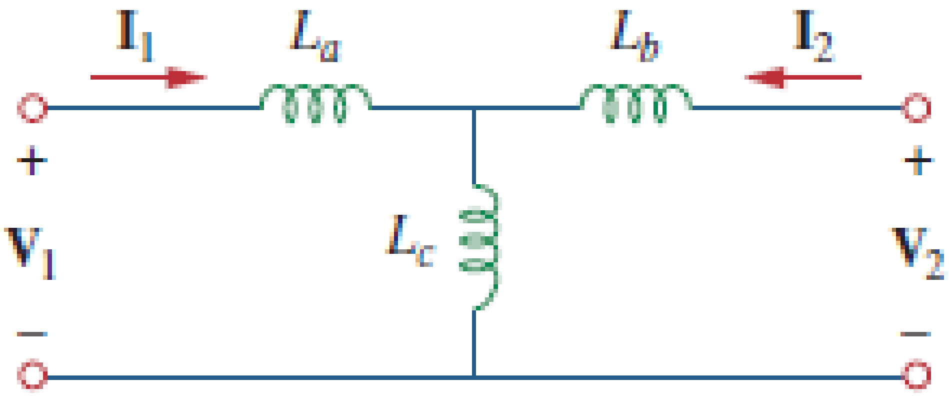
Linear Transformer

- The voltage equation of the equivalent circuit of the linear transformer:

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

Linear Transformer

- The equivalent circuit of T circuit.



Linear Transformer

- The voltage equation of the equivalent circuit of the T circuit:

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} j\omega(L_a + L_c) & j\omega L_c \\ j\omega L_c & j\omega(L_b + L_c) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

- For equivalent the terms in the two voltage equation must be equal, so that

$$\boxed{L_a = L_1 - M, \qquad L_b = L_2 - M, \qquad L_c = M}$$

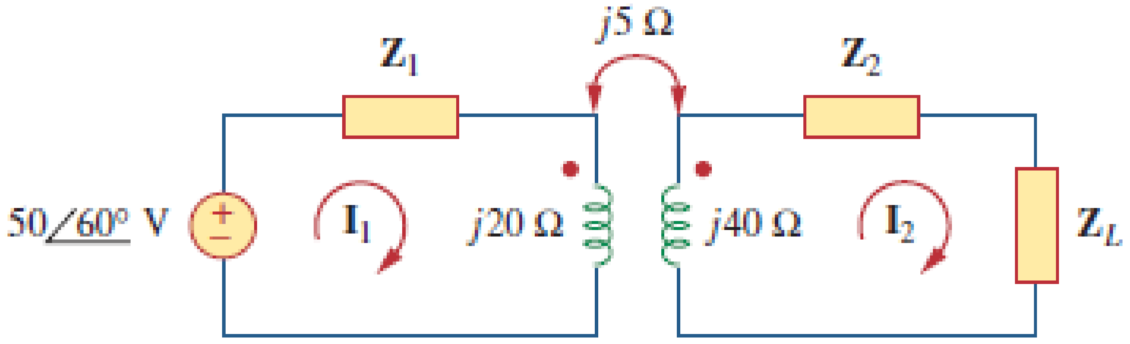
Linear Transformer

- The equivalent circuit of the linear transformer may be replace to $\pi\ circuit$.

assignment

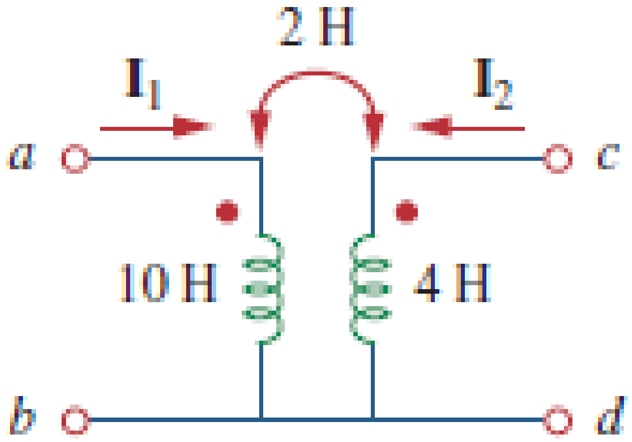
Linear Transformer

Example5: In the following circuit, calculate the input impedance and current **I**1. Take **Z**1= 60- *j*100 , **Z**2=30+*j*40 , and **Z***L*=80+*j*60 .



Linear Transformer

Example6: Determine the T-equivalent circuit of the linear transformer in the following circuit.



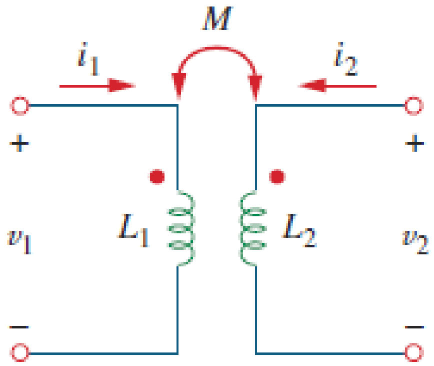
Ideal Transformers

- An ideal transformer is one with perfect coupling (K=1).
- Primary and secondary coils are lossless ($R_1=R_2=0$).
- Coils have very large reactance ($L_1, L_2, M \rightarrow \infty$).

An **ideal transformer** is a unity-coupled, lossless transformer in which the primary and secondary coils have infinite self-inductances.

Ideal Transformers

- The equivalent circuit of the transformer:



- The voltage equation of the primary side is:

$$\mathbf{V}_1 = j\omega L_1 \mathbf{I}_1 + j\omega M \mathbf{I}_2$$

Ideal Transformers

- The voltage equation of the secondary side is:

$$\mathbf{V}_2 = j\omega M \mathbf{I}_1 + j\omega L_2 \mathbf{I}_2$$

- From the two voltage equation:

$$\mathbf{V}_2 = j\omega L_2 \mathbf{I}_2 + \frac{M\mathbf{V}_1}{L_1} - \frac{j\omega M^2 \mathbf{I}_2}{L_1}$$

- But for ideal transformer k=1 (perfect coupling)

$$M = \sqrt{L_1 L_2}$$

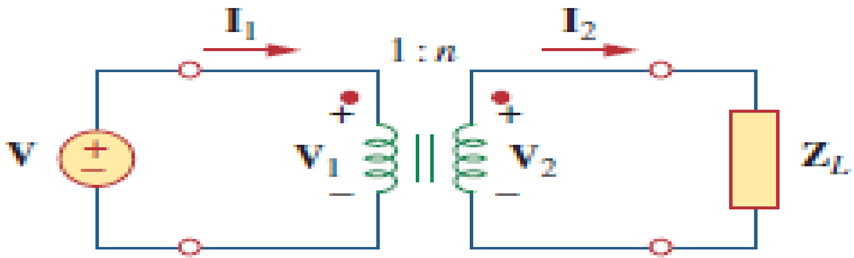
- Substitute with M in V2

Ideal Transformers

$$\mathbf{V}_2 = j\omega L_2 \mathbf{I}_2 + \frac{\sqrt{L_1 L_2}\mathbf{V}_1}{L_1} - \frac{j\omega L_1 L_2 \mathbf{I}_2}{L_1} = \sqrt{\frac{L_2}{L_1}}\mathbf{V}_1 = n\mathbf{V}_1$$

$$n = \sqrt{L_2/L_1}$$

- Where n is the turns ratio
- The equivalent circuit of ideal transformer is :



Ideal Transformers

- Because of the high permeability of the core, the flux links all the turns of both coils.
- The same magnetic flux goes through both windings. According to Faraday's law, the voltage across the primary and secondary winding is:

$$v_1 = N_1 \frac{d\phi}{dt}$$

$$v_2 = N_2 \frac{d\phi}{dt}$$

Ideal Transformers

$$\frac{v_2}{v_1} = \frac{N_2}{N_1} = n$$

- Due to the coils are lossless, the energy supplied to the primary must equal the energy absorbed by the secondary.

$$v_1 i_1 = v_2 i_2$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = n$$

Ideal Transformers

- Where, $n = 1 \leftrightarrow$ Isolation transformer
- Where, $n > 1 \leftrightarrow$ Step up transformer
- Where, $n < 1 \leftrightarrow$ Step down transformer

Ideal Transformers

- Check the ideal transformer is loss less:

$$\mathbf{V}_1 = \frac{\mathbf{V}_2}{n} \quad \text{or} \quad \mathbf{V}_2 = n\mathbf{V}_1$$

$$\mathbf{I}_1 = n\mathbf{I}_2 \quad \text{or} \quad \mathbf{I}_2 = \frac{\mathbf{I}_1}{n}$$

The complex power in the primary winding is

$$\boxed{\mathbf{S}_1 = \mathbf{V}_1\mathbf{I}_1^* = \frac{\mathbf{V}_2}{n}(n\mathbf{I}_2)^* = \mathbf{V}_2\mathbf{I}_2^* = \mathbf{S}_2}$$

- Showing that the complex power supplied to the primary is delivered to the secondary without loss. The transformer absorbs no power.

Ideal Transformers

- The input impedance :

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{1}{n^2}\frac{\mathbf{V}_2}{\mathbf{I}_2}$$

- It is evident from the equivalent circuit of the ideal transformer $Z_L = \frac{V_2}{I_2}$, so that

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{Z}_L}{n^2}$$

Ideal Transformers

- The dot rules for ideal transformer is stated as follows:

- ✓ If v_1 and v_2 are both positive or both negative at the dotted terminals, $\frac{v_2}{v_1} = n$, otherwise $\frac{v_2}{v_1} = -n$

- ✓ If i_1 and i_2 are both enter into or both leave from the dotted terminals, $\frac{i_1}{i_2} = -n$, otherwise $\frac{i_1}{i_2} = n$

Ideal Transformers

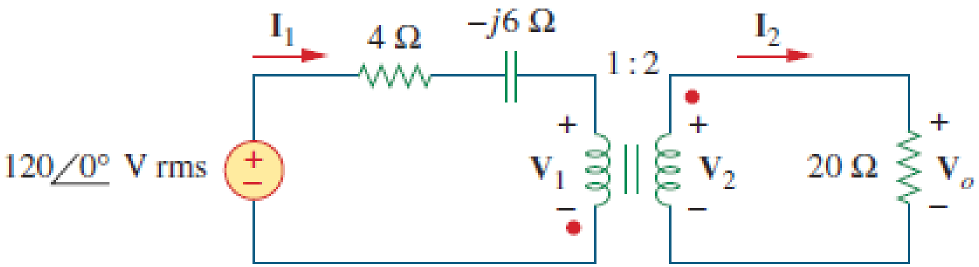
- Example7:

An ideal transformer is rated at 2400/120 V, 9.6 kVA, and has 50 turns on the secondary side. Calculate: (a) the turns ratio, (b) the number of turns on the primary side, and (c) the current ratings for the primary and secondary windings.

Ideal Transformers

➢ Example8:

For the ideal transformer circuit, find: (a) the source current $\mathbf{I}_1$, (b) the output voltage $\mathbf{V}_o$, and (c) the complex power supplied by the source.




THANK YOU